HYPER VERSION

Received: *January 8, 1998*, Accepted: *February 17, 1998*

Fractional branes and wrapped branes

Duiliu-Emanuel Diaconescu and Jaume Gomis
Department of Physics and Astronomy, Rutgers University
Piscataway, NJ 08855–0849
E-mail: duiliu@physics.rutgers.edu, jaume@physics.rutgers.edu

Michael R. Douglas
Department of Physics and Astronomy, Rutgers University
Piscataway, NJ 08855–0849
Institut des Hautes Études Scientifiques, Le Bois-Marie
Bures-sur-Yvette, 91440 France
E-mail: mrd@physics.rutgers.edu

Abstract: We discuss the "fractional D-branes" which arise in orbifold resolution. We argue that they arise as subsectors of the Coulomb branch of the quiver gauge theory used to describe both string theory D-brane and Matrix theory on an orbifold, and thus must form part of the full physical Hilbert space. We make further observations confirming their interpretation as wrapped membranes.

Keywords: Branes in String Theory, M(atrix) Theories.

JHEP02(1998)013

Contents

1 Introduction

D-branes propagating on resolved orbifolds provide a simple and tractable example of how geometry at short distances arises as the low energy configuration space of a world-volume gauge theory [1, 2, 3, 4, 5]. They also provide a simple and explicit starting point for the definition of Matrix theory [6] on these spaces [7, 8, 9, 10].

The construction (reviewed in section 2) starts with maximally supersymmetric gauge theory, to which a projection acting simultaneously on space-time and Chan-Paton indices is applied, leading to a 'quiver' gauge theory whose Higgs branch is the orbifold. In string theory, the twist fields which parameterize the blow-up of the orbifold control Fayet-Iliopoulos terms, which resolve the low energy configuration space into a smooth space.

In [3] it was pointed out that these theories, with zero Fayet-Iliopoulos terms, also have a Coulomb branch. This was interpreted as describing *wrapped* D-branes – if the original theory described Dp-branes, the Coulomb branch describes a collection of D$p+2$-branes wrapped about the various two-cycles which have shrunk to zero volume in the orbifold limit. The simplest reason to believe this is the following. The (classical) Coulomb branch is parameterized by scalar expectation values describing coordinates in the space transverse to the orbifold, and it has larger unbroken gauge symmetry: instead of $U(1)$ (for a single D-brane) it has unbroken $U(1)^n$ (for the A_{n-1} singularity with $n-1$ two-cycles). These parameters and gauge symmetry are consistent with an interpretation as n objects each bound to the fixed point in the orbifold, but free to move in the transverse dimensions.

In string theory, this interpretation can be confirmed by showing that the various objects are sources of the appropriate twisted sector RR fields, using the techniques of [1]. We describe this computation in section 2. From its form, it is clear that one can treat the objects independently by keeping an appropriate subsector of the gauge theory (e.g. a single $U(1)$), and that each has mass $1/n$ of the original D-brane, motivating the name 'fractional branes.' These masses agree with string theory expectations, arising from non-zero $\int B$ on the cycles.

From the space-time point of view, the wrapped membranes are the gauge bosons of ADE enhanced gauge symmetry (for $\mathbb{C}^2/\Gamma$) given masses by gauge symmetry breaking. In perturbative string theory, the gauge symmetry is always broken by $\int B$. In terms of the dual heterotic string theory, this can be thought of as a Wilson line A_{11}. One can also modify the gauge symmetry breaking by turning on additional Wilson lines; these correspond to turning on the FI terms. This will be shown more explicitly in section 3.

Turning on FI terms will change the masses of the gauge bosons, but they will still exist as BPS states in the theory. On the other hand, from the D-brane gauge theory point of view, the FI terms lift the Coulomb branch and remove these supersymmetric vacua! How can we reconcile these two statements?

The basic resolution is the observation that the Coulomb branch describes a state containing several charged gauge bosons, whose charges sum to zero. Such a state generally will not be BPS and thus the corresponding gauge theory state will not be a supersymmetric vacuum. We identify it as a metastable analog of the Coulomb branch but now with supersymmetry broken by the FI terms and vacuum energy ζ^2. The metastability corresponds to the possibility for the wrapped branes to annihilate. On the other hand, if their space-time separation is large, the dynamics involving one of the constituents is described by the fractional brane prescription. Thus cluster decomposition requires including the fractional branes. We develop this argument in detail in section 3 and resolve several related paradoxes there.

Following [7], we also claim that this description is also valid in Matrix theory. On a qualitative level, the justification is the same – since the identification of fractional branes with wrapped branes was justified by BPS arguments, it must survive at strong string coupling and thus in M theory. More recently a quantitative prescription for the Matrix theory limit has been given [11] and thus we can check whether these states survive in this limit. We do this in section 4, and find that the states do survive, essentially because they carry the appropriate D0-brane charge for a BPS bound state.

Section 5 contains conclusions.

2 Review of the construction

We consider type IIa string theory compactified on an ALE $\mathcal{M}$ asymptotic to $\mathbb{C}^2/\Gamma$, or equivalently M theory on $\mathcal{M} \times S^1$. This produces a six-dimensional theory containing maximal SYM with ADE gauge group G, as was first predicted by considerations of type IIa – heterotic duality. The origin of enhanced gauge symmetry is well-known in the language of eleven-dimensional supergravity – a two-brane wrapped around the two-cycle Σ of $\mathcal{M}$ produces a gauge multiplet charged under $\int_\Sigma C^{(3)}$. The ALE moduli ζ^i determine the complex structure and volume of the two-cycles and translate directly into the scalars

ϕ^i of the $d = 7$ SYM multiplet. The expectation values $\int_\Sigma B$ in IIa language become $\int_\Sigma C^{(3)} = A_{11}$ in M theory language.

In the substringy regime [2] an explicit construction of the ALE can be given as the moduli space of a D-brane world-volume theory defined on an orbifold. This description does not start from supergravity (rather, gravitational effects emerge) and thus geometric phenomena such as wrapped two-branes must find new explanations. However, since supergravity is explicitly described by the closed string sector, it is not hard to explicitly check these explanations against the original one.

The Lagrangian is the projection of maximally supersymmetric SYM with additional FI terms. We denote coordinates in the ALE as Z and transverse to the ALE (in $\mathbb{R}^5$) as X. The potential of the gauge theory is

$$V = \sum [X, Z]^2 + ([Z, Z] - \zeta)^2 \,. \tag{2.1}$$

As discussed in many places the branch $Z \neq 0$ has ALE topology and metric. On this branch the off-diagonal components of X and the gauge field are massive and thus this world-volume theory has the expected degrees of freedom of a single D-brane.

If $Z = \zeta = 0$ we can give a vacuum expectation value to the off-diagonal components of X. This is the Coulomb branch, with unbroken $U(1)^n$ gauge symmetry. The natural interpretation is n D0-branes free to move in the transverse dimensions, each with mass $1/ng_s l_s$. Each $U(1)$ gauge multiplet contains fermions whose zero modes act on an $8+8$-dimensional representation space and thus we must identify these particles as BPS multiplets in space-time. As described in [7], in the orbifold limit, the enhanced $SU(n)$ gauge symmetry of M theory compactification on the A_{n-1} singularity is broken by an explicit B-field vacuum expectation value, and the only candidate BPS multiplets are the massive gauge bosons of this broken symmetry.

The lower dimensional interpretation of the B-field vev is as a Wilson line A_{11} and so this interpretation requires the individual fractional branes to carry a specific non-zero eleven-dimensional momentum. In IIa string theory this is RR one-form charge so this can be tested by world-sheet computation. Furthermore, the space-time $U(1)^{n-1}$ arises from twisted sector RR fields, so these charges are also computable.

2.1 World-sheet computation

We briefly outline the computation of the one-point function of the relevant RR field vertex operators on a disk whose boundary is a fractional brane. For more detail than presented here, we refer to the appendix of [1] as well as to [12].

We first review the world-sheet computation for flat D-branes. The RR vertex operator, in a picture with superconformal ghost number -2 (appropriate for the disk), is

$$C_{\mu_1 \ldots \mu_n} \left(\Gamma^{\mu_1} \ldots \Gamma^{\mu_n}\right)_{\alpha\beta} e^{-\frac{3\phi(z)}{2}} e^{-\frac{\tilde{\phi}(\bar{z})}{2}} c(z)\tilde{c}(\bar{z}) S^\alpha(z) \tilde{S}^\beta(\bar{z}) \,. \tag{2.2}$$

The expectation value on the upper half plane can be computed by extending the fields on the complex plane using the boundary conditions

$$S^\alpha(z) = \begin{cases} S^\alpha(z), & z \in \mathcal{H}^+ \\ (\Gamma^0 \dots \Gamma^p)^\alpha_\beta \, \tilde{S}^{\dot\beta}(z), & z \in \mathcal{H}^- \end{cases}, \tag{2.3}$$

corresponding to $p+1$ Neumann conditions and $9-p$ Dirichlet boundary conditions. This leads straightforwardly to a non-zero expectation value for the operator corresponding to $C^{(p)}_{0\dots p}$.

On the orbifold and in the RR sector, the twist eliminates the fermion zero modes in the internal space, and the RR boundary state is a bispinor in $d = 6$. For a brane transverse to the orbifold, this leads to the same expectation value for any $C^{(p)}$, multiplied by the expectation value of the twist and Chan-Paton contributions.

The twist part of the vertex operator is simply $\sigma_g \tilde{\sigma}_g$ where σ_g produces the supersymmetric ground state in the sector twisted by the group element g. Again by extending the fields, the one-point function of this operator is 1. In correlators involving other operators the correlation functions with these twist fields can have cuts and this requires the further association of twist fields with a contribution to the trace over the boundary Chan-Paton factor [1]. This leads to the final amplitude $\mathcal{A} = \mathrm{Tr}\ \gamma_g$ where γ_g is the representation of g on the particular fractional brane of interest. For A_{n-1} singularities this will be a phase $\exp 2\pi i k/n$ for some k.

For $g = 1$ we find that the charge of a single fractional brane under the untwisted RR field (and under other untwisted fields, including the metric and dilaton) is $1/n$ times the contribution for the original (Higgs branch) D-brane. The charge for the g-twisted RR field is γ_g and changing basis to the twisted RR fields associated with particular nodes of the quiver diagram leads to a unit charge for the $U(1)$ associated with the k'th node. We interpret this as a brane wrapped around the two-cycle whose moduli is controlled by the NS-NS twist fields partner to this RR field.

Thus an individual fractional brane has the same charge as a wrapped two-brane and carries $p_{11} = 1/nR_{11}$. Furthermore, the analogous computation for the Coulomb branch of the original (regular representation) theory involves a sum of disk diagrams each with Dirichlet boundary conditions fixed to a particular X, so the fractional branes which appear in this way carry the same charges.

3 Blowing up the two-cycles

If $\zeta \neq 0$ the target space has ALE topology and metric. In this situation, the wrapped two-branes are still there, but we lift the Coulomb branch. What is their new description?

They must still be the same Coulomb branch, but now no longer supersymmetric vacua. This is clear by an adiabatic argument. One could first separate the fractional branes with $\zeta = 0$, then slowly turn on ζ. Since they are individually BPS states, they cannot decay. The resulting description of branes wrapped around two-cycles of volume ζ is the same Coulomb branch, but now lifted to a non-supersymmetric vacuum. Since we can produce n fractional branes from the vacuum, space-time cluster decomposition requires us to include the individual fractional brane sectors.

The effective theory is the same $U(1)^n$ gauge theory, but with a single additional term obtained by substituting $Z = 0$ into the potential (2.1):

$$\mathcal{L} = \sum_i \mathcal{L}_{U(1)_i} + \sum \zeta^2 . \tag{3.1}$$

Although the vacuum is not supersymmetric, the supersymmetry breaking takes the rather trivial form of a constant shift in the vacuum energy. This explains how a non-supersymmetric vacuum can still have the requisite $8n$ fermion zero modes for n BPS multiplets in space-time.

If we turn on individual X_i's to separate the fractional branes, to a good approximation we can just describe each one by its individual $\mathcal{L}_{U(1)_i}$ Lagrangian. This picture generalizes in an obvious way to charged states corresponding to non-simple roots and other bound states.

In general, in these theories with eight supersymmetries the Coulomb branch metric will get quantum corrections. These will correspond to long-range forces between the 2B and anti-2B.

All this might be expected, because the state obtained by moving out on the Coulomb branch has total G gauge charge zero, and one would be tempted to say that it cannot be BPS. However, this explanation is not complete, because it suggests that the Coulomb branch with $\zeta = 0$ also should not be a supersymmetric vacuum.

How can a state containing a wrapped 2B and anti-2B be BPS ? The answer is that these charges do not add to zero but rather add to the 0B charge. Let us write the relevant part of the charge vector as (Q_0, Q_2); then (consider the example of an A_1 singularity) the 0B has charge $(1, 0)$, and the 2B's have charge $(1/2, \pm 1)$. From the space-time point of view, the masses of these states are simply determined in terms of the $d = 6$ scalars ϕ as

$$m^2 = \sum_{i=1}^{4} (Q_0 \phi_0^i + Q_2 \phi_2^i)^2 . \tag{3.2}$$

The FI terms correspond to ϕ_2^i and as long as these are zero, all three of the states will be BPS, with non-zero mass determined by $\phi_0^i = \int B$ or equivalently the scalar from A_{11}. Call this non-zero component ϕ_0^4. Turning on the FI terms will make the Coulomb branch non-BPS with $\Delta m^2 \sim \zeta^2$ exactly if they control the three orthogonal scalars ϕ_2^i with $1 \leq i \leq 3$. This orthogonality can also be seen in the 'Narain lattice' description of the moduli space, motivated by type II–heterotic duality [13].

Thus the 'fractional branes' can also be thought of as D0–D2 bound states.

4 Matrix theory limit

In [7] it was proposed to define Matrix theory on an ALE using the same prescription, following the general arguments of [6] which obtained Matrix theory on flat space by boosting IIa string theory to the infinite momentum frame.

More recently it has been argued by Sen and by Seiberg that the rescalings done in a non-systematic way in [6] to derive their Hamiltonian in fact provide a recipe for producing Matrix theories in general backgrounds.

The recipe was applied to the case at hand in [10]. In general all of the structure in a D0-brane gauge theory Lagrangian survives this limit (indeed this scaling was first proposed for this very reason [14]). In particular, the form of the ALE metric is unaffected by the rescaling.

On the other hand, the relation for the other branes is not necessarily the same. In particular, the D2-branes of toroidal compactification of IIa theory disappear in the limit, to be replaced by BPS bound states of the D0 and D2-branes described by magnetic fluxes in the Matrix gauge theory.

Are 'fractional branes' more like D0-branes or like D2-branes from this perspective? From the previous discussion, they are BPS states carrying both D0 and D2-brane charges, and like other such bound states would be expected to survive in Matrix theory.

It was checked in [7, 5, 9] that the light-cone energy P_+ of these states stays finite in the limit. The ALE Matrix theory is $U(N)^n$ quantum mechanics with field content specified by an A_{n-1} quiver diagram, and the careful analysis of the quiver gauge theory charges in [9] leads to the results

$$\begin{aligned} P_+ &= \frac{nR_{11}}{2N}\frac{\zeta^2}{(2\pi)^2 {l_p}^6}\,, \\ P_- &= \frac{N}{nR_{11}}\,. \end{aligned} \tag{4.1}$$

We see that P_+ is given by the the expected wrapping contribution to the energy.

5 Conclusions

We see that the D-brane description of ALE space does contain the expected BPS states, and realizes the enhanced gauge symmetry of M theory compactification in the manifest form claimed in [7].

An interesting problem for future research would be to find the annihilation cross section for a pair of oppositely wrapped D2-branes to annihilate into a D0-brane, a problem which has not yet been solved for unwrapped D2-branes. A naive extrapolation of the weakly coupled heterotic string amplitude $O(g_{het})$ would lead to a diverging amplitude at weak type II coupling, so it seems very likely that this is a "BPS algebra structure constant" [15] with explicit dependence on the moduli.

In this description, it is determined by the transition amplitude from the Coulomb to the Higgs branch in a quantum mechanics with finitely many degrees of freedom. Given energy $E > \zeta^2$, the opposite process of pair creation of wrapped branes is classically allowed [2], and it is not at all obvious that such amplitudes will be suppressed in weak string coupling.

If we compactify Matrix theory on a further S^1, we get a $1+1$-dimensional version of the theory, which has been argued [16] to exhibit decoupling of Coulomb and Higgs branches.

Acknowledgments

We thank P. Aspinwall, S. Lukyanov, J. Maldacena, S. Shenker and A. Zamolodchikov for discussions.

Note. This work was largely done in September, as preparation for studying the question described in the conclusions. Recently a paper has appeared which disagrees with the basic premise of this work [17], prompting us to publish our results so far.

References

[1] M.R. Douglas and G. Moore, hep-th/9603167.

[2] M.R. Douglas, D. Kabat, P. Pouliot and S. Shenker, *Nucl. Phys.* **B 485** (1997) 85 [hep-th/9608024].

[3] J. Polchinski, *Phys. Rev.* **D 55** (1997) 6423 [hep-th/9606165].

[4] C. Johnson and R. Myers, *Phys. Rev.* **D 55** (1997) 6382 [hep-th/9610140].

[5] M.R. Douglas, B. Greene and D.R. Morrison, *Nucl. Phys.* **B 506** (1997) 84 [hep-th/9704151].

[6] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[7] M.R. Douglas, **JHEP 07** (1997) 004 [hep-th/9612126].

[8] M.R. Douglas, H. Ooguri and S. H. Shenker, *Phys. Lett.* **B 402** (1997) 36-42 [hep-th/9702203].

[9] D.-E. Diaconescu and J. Gomis, hep-th/9707019.

[10] M.R. Douglas and H. Ooguri, hep-th/9710178.

[11] A. Sen, hep-th/9709220;
N. Seiberg, *Phys. Rev. Lett.* **79** (1997) 3577 [hep-th/9710009].

[12] A. Hashimoto, I.R. Klebanov, *Nucl. Phys.* **B 55** *(Proc. Suppl.)* (1997) 118 [hep-th/9611214].

[13] P.S. Aspinwall, TASI 96, hep-th/9611137.

[14] D. Kabat and P. Pouliot, *Phys. Rev. Lett.* **77** (1997) 1004 [hep-th/9603127]; U.H. Danielsson, G. Ferretti and B. Sundborg, *Int. J. Mod. Phys.* **A 11** (1996) 5463 [hep-th/9603081].

[15] J. Harvey and G. Moore, hep-th/9609017.

[16] E. Witten, **JHEP 07** (1997) 003 [hep-th/9707093].

[17] D. Berenstein, R. Corrado and J. Distler, hep-th/9712049.